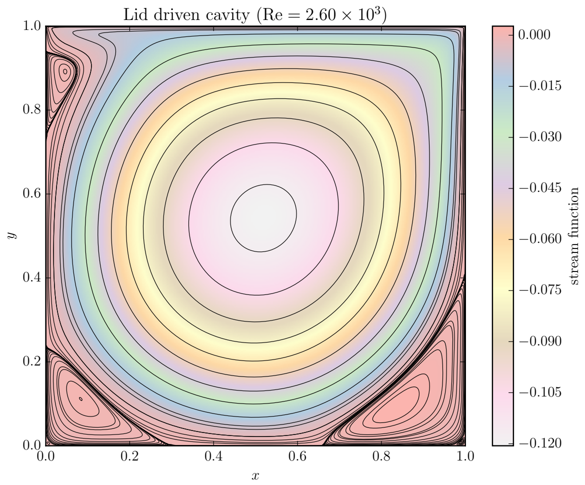
Numerical Methods



— OBJECTIVES

In physics, the mathematical description of many phenomenons often involves the resolution of differential equations, either Ordinary Differential Equation (ODE) or Partial Differential Equation (PDE). In astrophysics the need for numerical simulations is critical since the objects under consideration cannot be studied in the lab. A large variety of physical problems are described in that way: fluid dynamics, heat conduction, n-body simulation, radiative transfer, etc . . . Solving the equations of mathematical physics numerically will be our playground to face the classical problems of numerical analysis such as interpolating, integrating, solving linear system of equations, etc . . . and on the way learn how to program efficiently. The student will learn how to develop and study the behaviour of various numerical schemes, implement them in the form of computer programs and used them to solve the classical equations of mathematical physics.

— EVALUATION

- Homework: 10%
- Numerical projects: 50%
- End-of-term exam: 40%

— MAIN PROGRESSION STEPS

- **First half**: Courses and exercises
- **Second half**: Numerical project and practical works

— BIBLIOGRAPHY & RESOURCES

- A First Course in Numerical Analysis, Ralston, A. and Rabinowitz, P., Dover Publications, Inc., 2001
- Finite difference methods for differential equations, LeVeque, R.J., 2007
- Finite difference and spectral methods for ordinary and partial differential equations, Trefethen, L.N., 1996
- Numerical methods for engineers and scientists, Hoffman, Joe D and Frankel, Steven, CRC press, 2001
- Finite volume methods for hyperbolic problems, LeVeque, Randall J, Cambridge university press, 2002

— CONTACT

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Content

— Part. 0 - Computer skill sharpening

by G. Niccolini

- The `GNU/Linux` system
- Basic terminal commands
- The text editor: one tool to rule them all.
- Software management (`apt`, `snap`, installation from source)
- A touch of `bash` programming
- A short intro to `C++`, `python` and `LaTeX`

— Part. 1 - Fundamentals

by G. Niccolini

- Round-off error and truncation errors.
- Interpolation
- Discrete Fourier Transform (DFT)
- Integration of functions
  - Order of accuracy
  - Newton-Cotes formulae

  - Gaussian quadratures
- Solving Linear systems of equations
  - Iterative methods for sparse systems (Jacobi, Gauss-Seidel, Successive Over-Relaxation, conjugate gradient, ...)
- Finite Difference method
  - How to derive FD formulae
  - Truncation errors

— Part. 2 - Partial/Ordinary Differential Equations —

by G. Niccolini

- Numerical solution of ODEs
  - Solving differential equation numerically
    * Mathematical properties
    * One-step methods (Euler, RK, Taylor series, explicit vs implicit methods)
    * Multi-step methods (Leap-Frog, Adams-Moulton, Adams-Bashforth, ...)
    * Stability analysis : notion of zero and absolute stability, characteristic polynomials
  - Solving PDEs
    * Mathematical properties
    * PDEs as vector ODEs
    * Boundary conditions
    * Truncation error and consistency
    * Convergence and stability
    * Von Neumann stability criterion

— Part. 3 - Practical works and project —

by G. Niccolini

- Introduction to programming: `C++` and `python`
- Development of computer programs to solve linear classical differential equations of mathematical physics (wave equation, Schrödinger, diffusion, advection, n-body problems, etc ...)